# Quantum Field Theory for Philosophers

The Quantum Theory of Fields in the Contemporary Locus of Metaphysical Research

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## Introduction

QFT has emerged during last 10 years as important framework in which to do elementary particle physics

Are philosophers prepared to use current physical theory as a guide to resolving metaphysical questions?

Object paper is to tell straightforwardly what QFT has to say about 'reality'.

I am not going to deal with the interpretative problems of QM as such

## The Classical Concept of Field

Different approaches to theory of matter
Nature of forces between 'pieces' of matter.
↳ action-at-distance v. field theories

Field Theory — Associates certain properties with ~~points~~ space-time points

Eg E. m-field, Eulerian hydrodynamics

Particle Theory — Attributes to certain individuals (the particles) a variety of properties.

These properties will include space-time locations

What do we mean by an individual?
In classical physics a simplistic Lockean view is normally used.
Individuation transcends the properties of an entity — unknowable substratum — Transcendental Individuality (TI).

But in classical particle physics spatio-temporal continuity of trajectory can be used (cf. Boltzmann (1897))

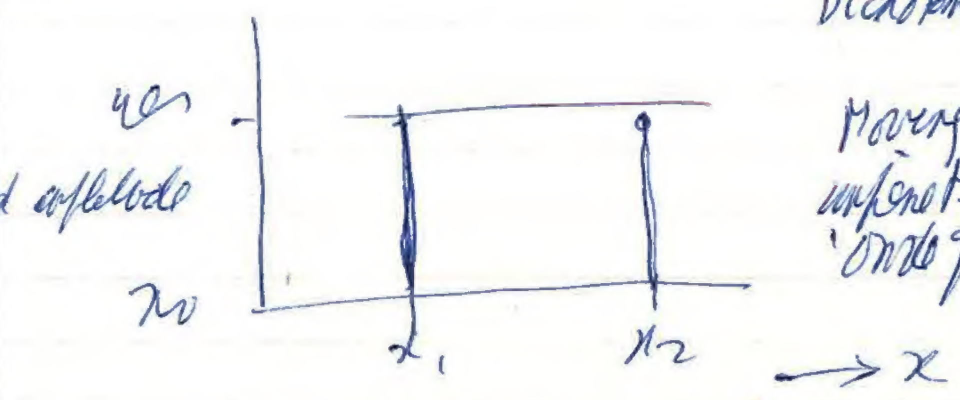
This is not available in QM and if elementary particles are to be regarded as individuals TI must be assumed.
This will be used as an argument against particle approaches to the interpretation of QFT.

Query: Do space-time points possess TI?

Basic difficulty for field theories of matter
How do objects get individuated?

Field Approach to classical particle physics

Dichotomic yes/no field.




Moving property of impenetrability — Newton 'On the Gravity & Equilibrium of Fluids'.

Underdetermination as between particle and field approaches — Arrogance (no matter of fact at stake) (Newton-Smith) v. Ignorance (matters of fact not decidable by observation)

# History of classical field Theories

1. Field theories of matter in 18th C. (Bruton, Priestley) of Faraday "A Speculation touching ... the Nature of Matter", 1844.

2. Reduction of force to particles, effluvial theories

3. 'Wave theories' of force – propagation of a state of stress in a medium.
→ 19th C. Ether theories

4. Reduction of matter to ether (Thomson vortex atom) — Larmor

5. Lorentzian dualism: charged particles and substantial, but not mechanical ether.
↳ Energy as new ontological category
↳ electromagnetic theories of matter
Abraham, Wien → Mie, Born, Infeld.

6. Einstein – substantial nature of energy via $E = mc^2$ and Equivalence Principle

G.R. → metric tensor field
↳ Unified theories of gravitation & e.m.
matter = local concentration of geometrical fields
↓ geometrodynamics of Wheeler.

# Quantum Field Theory

## Two main approaches

Field Quantization

1. Classical field is regarded as a 'mechanical' system with an $\infty$ no. degrees of freedom — subjected to canonical quantization field amplitudes $\longrightarrow$ operator.

Ex Real Klein-Gordon field

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \mu^2\right)\psi = 0.$$

$\psi$ is 'configuration' field

Define canonically conjugate 'momentum' field $\Pi = \frac{\partial \psi}{\partial t}$.

Total energy of field $\mathcal{H} = \int H \, d^3\underline{r}$

where $H = \frac{1}{2}\left\{\Pi^2 + c^2|\nabla\psi|^2 + c^2\mu^2\psi^2\right\}$

Fourier analyze the 'configuration' and 'momentum' fields

$$\mathcal{H} = \frac{1}{2}\sum_{\underline{k}}\left(p_{\underline{k}}^+ p_{\underline{k}} + \omega_{\underline{k}}^2 \, q_{\underline{k}}^+ q_{\underline{k}}\right)$$

$$\omega_{\underline{k}} = c\sqrt{\mu^2 + k^2}.$$

Define
$$a_{\underline{k}} = 1/\sqrt{2\hbar\omega_{\underline{k}}}\left(\omega_{\underline{k}} q_{\underline{k}} + i p_{\underline{k}}^+\right)$$
$$a_{\underline{k}}^+ = \frac{1}{\sqrt{2\hbar\omega_{\underline{k}}}}\left(\omega_{\underline{k}} q_{\underline{k}}^+ - i p_{\underline{k}}\right)$$

then
$$\left.\begin{aligned}[a_{\underline{k}}, a_{\underline{k}'}^+] &= \delta_{\underline{k},\underline{k}'} \\ [a_{\underline{k}}, a_{\underline{k}'}] &= [a_{\underline{k}}^+, a_{\underline{k}'}^+] = 0\end{aligned}\right\}$$

$N_{\underline{k}} = a_{\underline{k}}^+ a_{\underline{k}}$ has eigenvalues $0, 1, 2 \cdots$

$$\left.\begin{array}{l} H = \hbar \sum_{\underline{k}} \left(N_{\underline{k}} + \tfrac{1}{2}\right) \omega_{\underline{k}} \\ \underline{P} = \hbar \sum_{\underline{k}} N_{\underline{k}}\, \underline{k} \end{array}\right\}$$

Eigenvalues of $H$ & $P$ are

$$E = \sum_{\underline{k}} n_{\underline{k}} (\hbar \omega_{\underline{k}}) + \text{Const.}$$

$$\underline{P} = \sum_{\underline{k}} n_{\underline{k}}\, \underline{k}$$

where Const. $= \frac{1}{2} \sum_{\underline{k}} (\hbar \omega_{\underline{k}})$.

So the number of quanta present with momentum $(\hbar \underline{k})$ and energy $(\hbar \omega_{\underline{k}})$ in the particle representation is just the excitation number $n_{\underline{k}}$ of the $\underline{k}$-mode.

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Second-quantization

2. Start with $N$-particle Schrodinger Eq. for assembly of bosons – state vector totally symmetric under permutation of particle labels. – specified by giving no. $n_i$ of particles in $i$-th 1-particle state $|\psi_i\rangle$ (associated with energy $E_i$)

Then $E = \sum n_i E_i$

cf energy assemblage of harmonic oscillators

* is $E = \sum_i \left(n_i + \frac{1}{2}\right) E_i$ if $\omega_i = E_i/\hbar$.

But $\psi$ is what we would get by subjecting the 1-particle S. Eq. to a second-quantization

i.e. interpreting it as a ~~field~~ classical field equation and subjecting it to Field Quantization

But second-quantization is more general than the N-Particle Schrödinger equation because of constraint $\sum_i n_i = N$.

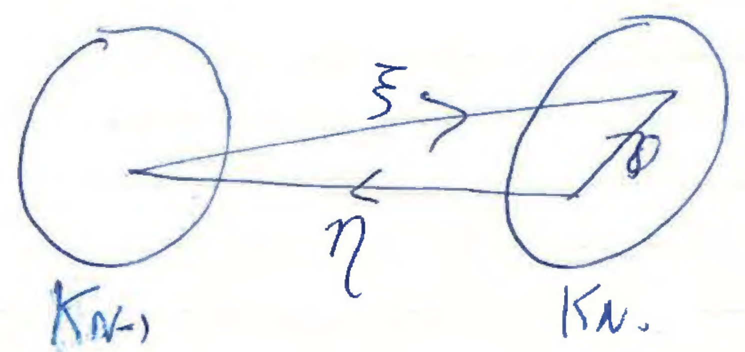
Fock Space $\mathcal{J} = K_0 \oplus K_1 \oplus \cdots \oplus K_N \oplus \cdots$

$K_0 \downarrow$ vacuum.

Creation and annihilation operators

$$a_i |n_i\rangle = \sqrt{n_i}\, |n_i - 1\rangle$$
$$a_i^+ |n_i\rangle = \sqrt{n_i + 1}\, |n_i + 1\rangle .$$

$Q = \xi \eta$




But $Q' = \xi + \eta$ would create and annihilate particles

N.B. Consider state
$$a_1^+ a_2^+ \ldots a_N^+ \, |\Phi_0\rangle \quad \rightarrow \text{vacuum}.$$
this is symmetric under permutation of state labels due to the commutation of the $a_i$'s.
This is what corresponds in 2nd quantized formulation to symmetry under permutation of particle labels in the N-particle formulation.

So 'Real' field $\xrightarrow{\text{Field quantization}}$ Quantum Field

N-particle S.Eq $\xrightarrow[\text{Second quantization}]{}$

Query But is Quantum Field the same animal in the 2 cases?

In particular for fermions 2nd quantization uses anticommutator brackets $\{a_i, a_j^+\} = \delta_{ij}$ etc

Response 1.) K. G. field real
S. field complex }

But complex field $\equiv$ pair of real fields
describes charged particles in RQFT
$\hookrightarrow$ notion of antiparticle
(nothing to do with −ve energy states.

2.) Classical limit of boson field is classical field
Classical limit of Fermion field is collection of particles.

But historical accident that QFT arrived at by canonical quantization imposed on a classical limit — does not mean different interpretation when we do not pass to the limit.

3.) Massless fields are 'field' theories
Massive fields are 'particle' " }

Distinction based on claimed nonlocalizability of massless quanta.
1st quantization: no position operator — Newton-Wigner (but cf Sands, Penin)
2nd quantization: particle production of localization with Compton wavelength $\hbar/mc$ ($\to \infty$ as $m \to 0$)

But does easy production of soft photons count against the localization of a hard photon ray?

4.) Weinberg (1964) approach to field theories via: creation/annihilation operators for canonical states + change of basis to get transformation properties under finite-dimensional nonunitary representations of the Lorentz group. In general the fields constructed by weinberg do not satisfy relativistic wave equations (other than trivial K. G. equation)

5.) Argument for field having ontological significance is based on causality condition

$$[Q(x), Q'(y)] = 0 \quad \text{if } (x-y)^2 < 0 .$$

(N.B. local observables actually averages over small space-time regions)

Violation would change statistics at x by performing measurement at y. (Contrast with EPR. nonlocality)

If Q's always contain an even number of Fermion field factors then

Sufficient condition for locality is for fields to commute or anticommute at space-like intervals

Spin - Statistics Theorem

Boson fields - spin of particles are integral or zero
Fermion fields - spin of particles half-integral

This theorem rules out anticommutation for ~~Boson fields~~ integral spin and commutation for half-integral spin.
Only leads to benign computation prescription if commutation / anticommutation are only possible.

Confine work of Green (1953) on Parafields

$$[a_k, \{a_l^{\dagger}, a_m\}] = 2\delta_{kl} a_m .$$

Trilinear commutation rules. for para boson fields
fundamental generalization to para fermion fields

Green Criterion for field quantization

Most concise equation of motion for field in terms of Hamiltonian $H$

$$i\hbar \frac{\partial \psi}{\partial t} = i\hbar [H, \psi].$$

Generalized Spin-Statistics Theorem (Dell'Antonio, Greenberg & Sudarshan 1964)

Excludes Parabosons for half-integer spins and Parafermions for integral spins

So Locality does not delimit the QFT as strongly as is sometimes supposed

N.B. Greenberg (1964) suggested quarks are parafermions (of order 3)

Creation and Annihilation Operators in Classical Mechanics

Commutation relations of creation/annihilation operators do not involve $\hbar$! — Description also available in classical mechanics.

Consider 'spike' fields as limit of continuous distribution $\rho(p_i, q_i, t)$
Eq. of motion for $\rho$

is $\rho(t) = e^{-iL(t-t_0)}\rho(t_0)$

where $L$ is Liouville operator

$$L = \sum_i \left(-i \frac{\partial H}{\partial p_i} \frac{\partial}{\partial q_i} + i \frac{\partial H}{\partial q_i} \frac{\partial}{\partial p_i}\right)$$

Consider motion of a single particle in phase space

Now factorize $e^{-iL(t-t_0)} = \xi \cdot \eta$

creator annihilator destroyer

$\eta$ annihilates initial state → vacuum
$\xi$ recreates final state from the vacuum.

Interpolate denumerable infinity of instants
$t_1, t_2 \ldots$ between $t_0, t$.
Particle is destroyed at each instant
and recreated at successive instant.

cf. doctrine of the Kalam espoused
by the Mutakallemim.

Accidents do not persist across
time atoms, but continually destroyed
and re-created.
God refrains from the decision
not to recreate.

N.B. Particle creation really only accepted
until Fermi's (1933) theory of β-decay.
(cf hole theory of positron and infinite sea
theory of photons (Dirac 1927)).

## Wave-Particle Duality

By equivalence result any phenomenon
can be described either by a (quantized)
particle theory or a (quantized) wave theory
— cf. Complementarity of wave & particle aspects.

Quote from Dirac (1927), refers to
'a complete harmony between the wave and light-quantum descriptions of the interaction [between atom and electro-magnetic waves]'.

But complementarity connected in QM with non-commutativity of operators.

Wave picture sharp $p$ (or $q$?)
Particle " sharp $q$ (or $p$?)

As in all discussions of complementarity there is good deal of uncertainty in what we are talking about!

QFT Particle representation has $N_k$ diagonal.

But $N_k$ does not commute with $\psi(\underline{x})$
$\psi(\underline{x},t)$ diagonal for all $\underline{x}$ (at a given time) is wave representation

But for complex field $\psi$ is not itself observable, but wave representation can given by Q diagonal for local observables Q (such as energy density)

<u>Query</u>: For interacting fields do both fields <u>have</u> to be quantized?
cp. Brown and Redhead (1981).

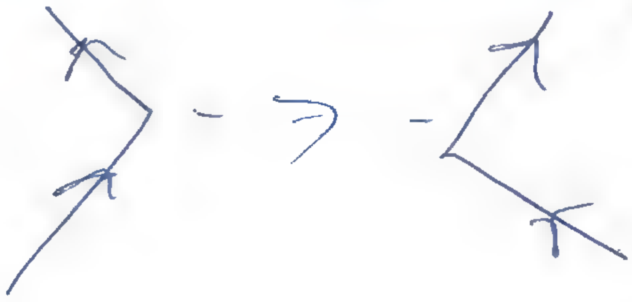
# Matter Fields and Force Fields – Unification

So far we have talked about non-interacting fields

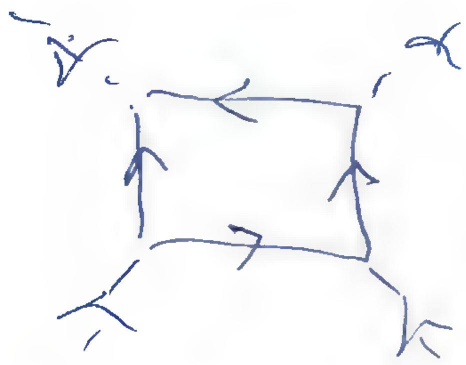
QFT describes interaction between matter quanta (particles) in terms of exchange of 'force' quanta, e.g. electromagnetic interactions described in terms of exchange of photons (actually virtual photons – see later).

But interactions of photons can also be regarded as mediated by electrons/positrons

cf.



with



So which is the matter particle and which is the force particle?

cf. Bootstrap approach to particle democracy in S-Matrix programme. Every particle plays 3 roles, constituent, composite and force particle!

Gauge Theories have restored distinction between Matter & Force
↓ Fermion fields ↓ Boson fields

but what is status of Higgs's field used to treat the symmetry of the vacuum?

Example. Gauge theories, ~~exhibit~~ $\psi$ satisfies Dirac Eq. Lagrangian density invariant under global phase transformation $\psi \to \psi e^{i\alpha}$.
But if we impose invariance under (generalized) local phase transformation $\psi \to \psi e^{i\alpha(x)}$.
This requires ~~intro~~ coupling to a new gauge field. Minimal choice is Maxwellian photon field.
So gauge invariance 'explains' the photon.

This distinction between fermion and boson fields has been blurred in supersymmetry theories $\to$ unified theory of all the interactions is Extended Supergravity — Also achieves Unification of Matter and Force

What do we mean by Unification?

One kind of entity enters into the unifying theory where previous phenomenal explanations in terms of distinct independent entities.

Eg (1) optical, electromagnetic action identified
(2) $\underline{E}$ and $\underline{H}$ combined into 6-component field tensor. Under proper Lorentz transformation $\underline{E}$, $\underline{H}$ have no separate covariant significance
(3) Gauge theory unification: Different kinds of particle identified as different states of the same entity
e.g. neutron, proton as two states of the nucleon.
But this is form-de Broglie, since gauge transformations have only an 'active' interpretation: one particle changed into another rather

than merely redescribed

Query Are gauge symmetries related to ontological economy?

(But note that 'natural kind' is itself philosophically controversial)

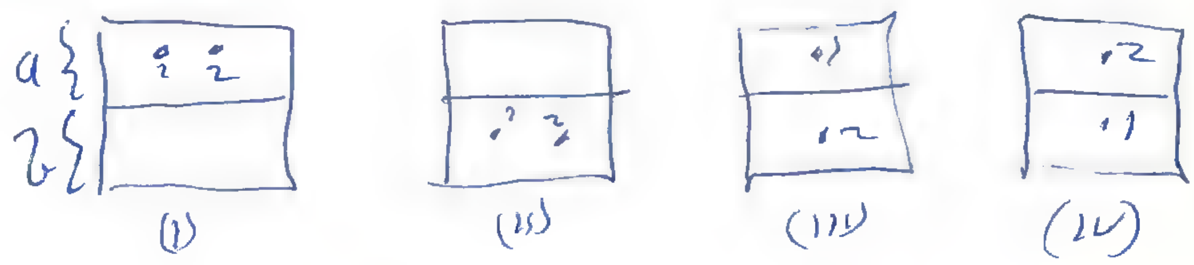
## The Problem of Individuality

Claim is often made that elementary particles do not possess TI → not individuals

Argument based on statistical mechanics of bosons & fermions

Consider possible states available to 2 indistinguishable particles distributed among 2 distinct 1-particle states

Classical statistical mechanics (assuming TI) gives the following possible distinct arrangements.




But in quantum statistical mechanics (iii) and (iv) are regarded as one of the same state in assigning statistical weights. This is taken as showing particles do not possess TI. The argument is as follows.

Quantum stat. Mech. can be done using the 1st quantized version of N-particle equation in which labels are attached to the particles

Consider the 4 possible product wave functions

$$\psi_a(\Lambda_1)\cdot\psi_a(\Lambda_2) \qquad \mathrm{I}$$
$$\psi_2(\Lambda_1)\cdot\psi_2(\Lambda_2) \qquad \mathrm{II}$$
$$\psi_a(\Lambda_1)\cdot\psi_2(\Lambda_2) \qquad \mathrm{III}$$
$$\psi_a(\Lambda_2)\cdot\psi_2(\Lambda_1) \qquad \mathrm{IV}$$

4-dimensional vector space equally well spanned by

$$\psi_a(\Lambda_1)\cdot\psi_a(\Lambda_2) \qquad \mathrm{V}$$
$$\psi_2(\Lambda_1)\cdot\psi_2(\Lambda_2) \qquad \mathrm{VI}$$
$$\frac{1}{\sqrt{2}}\left(\psi_a(\Lambda_1)\cdot\psi_2(\Lambda_2)+\psi_a(\Lambda_2)\,\psi_2(\Lambda_1)\right) \qquad \mathrm{VII}$$
$$\frac{1}{\sqrt{2}}\left(\psi_a(\Lambda_1)\cdot\psi_2(\Lambda_2)-\psi_a(\Lambda_2)\,\psi_2(\Lambda_1)\right) \qquad \mathrm{VIII}$$

symmetric (S) — V, VI, VII

antisymmetric (A) — VIII

Now for time evolution under a symmetric hamilton symmetry character of wave function is retained.

So if we impose as initial condition S or A then only one of the 2 states VII, VIII is ever available to the system. thus in the stat. weight attaching to the pair of states VII VIII gets halved.

The statistical weight in Q. Stat. Mech. can be regarded as reflecting dynamical restrictions on the accessibility of certain states.

N.B. Indistinguishability $\Rightarrow$ observables are symmetric functions of particle labels.

## The Indistinguishability Principle (IP)

Two particles are indistinguishable if, for any observable $Q$ ~~and~~ any label permutation $P$, and any state $\phi$

$$* \quad \langle P\phi|Q|P\phi\rangle = \langle \phi|Q|\phi\rangle$$

$$(\,)|P\phi\rangle \underset{def}{=} P|\phi\rangle.$$

But $\langle P\phi|Q|P\phi\rangle = \langle\phi|P^{-1}QP|\phi\rangle$ and $P$ is unitary.

$\Rightarrow$ $QP = PQ$, i.e. $P$ commutes with $Q$, so $Q$ is symmetric operator.

So indistinguishability imposes a restriction on observables. $\longrightarrow$ Parastatistics – Messiah & Greenberg (1965)

Note that $P|\phi\rangle = \pm|\phi\rangle$ is a sufficient condition for $*$ to hold, whatever the symmetry properties of $Q$.

This is the older sense in which IP was interpreted as a restriction on states led to bosons & fermions as only possibilities.

Connection between paraparticles & parafields

Stolt & Taylor (1970a) classified paraparticles into 3 types (1) parabosons finite order
(2) parafermions
(3) paraparticles infinite order

Stolt & Taylor (1970b) showed formal equivalence (1) and 2) with Green's parafields. Field theory of paraparticles is closer connected to be possible.

So we (probably) have situation of undetermination as between particle & field approaches to indistinguishable particles.

N.B. In classical mechanics (in field version for example) we can also individuate particles in terms of spatio-temporal continuity of trajectory (of the spikes). This approach is not available in QFT due to overlapping wave functions of 1-particle states. For very dilute gas where overlap can be neglected Q. St. Mech equivalent to classical Boltzmann results anyway.

## Vacuum and Virtual Particles

So far we have argued for underdetermination as between field & particle approaches.

But what about vacuum state $|\Phi_0\rangle$ — no particles present. Is $|\Phi_0\rangle$ a state of non-being, of complete quiescence.

But on field approach there are vacuum fluctuations in local observables ~~which are not diagonal~~ for which the no-particle state is not an eigenstate.

Cf ground-state of harmonic oscillator for which $\sigma = \sqrt{\langle q^2\rangle - (\langle q\rangle)^2} = \sqrt{\frac{1}{2\omega}}$ for angular frequency $\omega$.

1) Vacuum fluctuations are responsible for zero-point energy.

2) Vac. fluctuations explain, e.g., Lamb shift & anomalous magnetic moment of the electron, e.g. (Welton (1948) Koba (1949))

3) Casimir effect (1948) experimentally confirmed by Sparnaay (1958)
Attractive force between uncharged conducting plates — zero-point energy effect.

But all these results can be explained on an extended particle interpretation ~~which allows for quantities which are~~ not diagonal in the particle representation via the Fock-space machinery.

Virtual particles. Consider vacuum $|\Phi_0'\rangle$ of coupled fields — least eigenstate of $H_0 + H_I \to$ interaction Hamiltonian

$|\Phi_0'\rangle$ can be expanded as superposition of $|\Phi_0\rangle$ and appropriate many-particle eigenstates of $H_0$. These are called virtual particles. $|\Phi_0'\rangle$ contains no real particles, only virtual particles.

Similarly virtual particles ~~are~~ correspond to internal lines of Feynman diagrams; external lines correspond to real (bare) particles.

4-momentum conserved at each vertex but ~~not at~~ virtual particles are not on the mass-shell defined by $p_4^2 - c^2(p_1^2+p_2^2+p_3^2) = m^2c^4$.

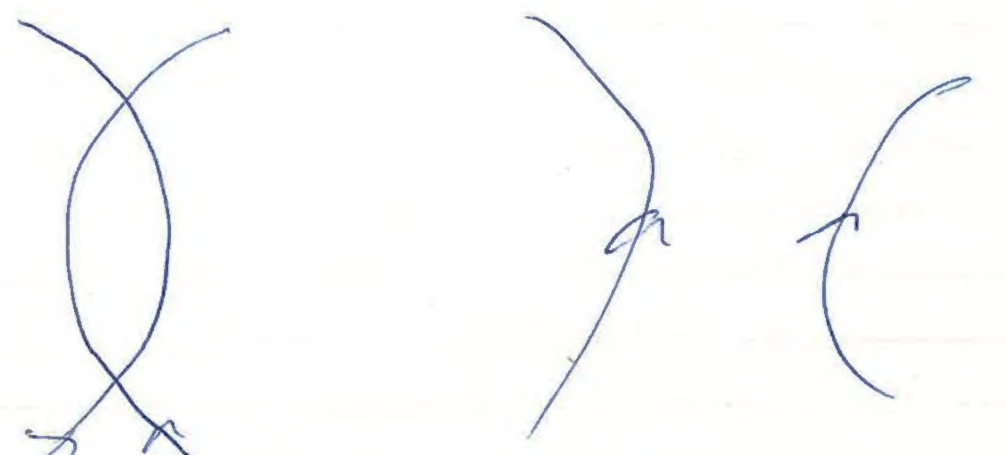
cf older non-covariant perturbation theory in which virtual particles were on the mass-shell but did not conserve energy in their creation and annihilation.

(v) Virtual particles arise in solving a problem specified by $H_0 + H'$ in terms of solutions of problem specified by $H_0$ alone.

N.B. no direct connection with vacuum fluctuations which occur in the absence of interactions.

Why does exchange of virtual particles not always produce repulsion?

Because in momentum space we cannot tell on which side particles pass each other



Attraction Repulsion

Indeed condition for well-defined scattering is that uncertainty in transverse location >> range of force.

# Conclusion

1.) Continuants reidentified in terms of TI

Ephemerals Distinguishable at a given instant of time but reidentifiable only if spatio-temporal continuity applies — cf moving wave packet.
— but not particle grain!

But A collection of particles is a single ephemeral — this is sense in which elementary particles are bereft of individuality

Ephemerals can be created and destroyed without the awkward notion of starting and stopping a continuant.

2) Particles as labelled individuals only possible if TI assumed, provided QM state-description is taken as complete.

But if positions for example of individual electrons are allowed as hidden parameters TI might not have to be assumed.

NB. For bosons and fermions ~~at~~ 2 particles always are in same state in sense that each partakes of each 1-particle state with equal probabilistic weight. This Assures 1-particle magnitudes are ontologically available, although not as observables.
But for higher-order many particle this is not true unless we restrict {physical magnitudes} to {observables}.

So any philosophical arguments against TI (do not know what it means e.g.) will count against particle approach to QFT.

3) Heuristic Role of field approach

Emphasis is on local quantities — both very artificial from particle point of view, not being diagonal in the particle representation.

Developments in QFT have consisted in taking the field programme seriously and letting it lead on in directions which are natural within that programme (e.g. self-energy effects, vacuum polarization etc).

4) Ambivalent attitude to QFT in 1930's due to infinities
↳ patched up in renormalization programme in late 1940s.

But in getting interactions nothing could be calculated, and perturbation theory did not apply

and in weak interactions Fermi theory not renormalizable.

Only major success QED

Development of gauge theories has led to renormalizable theory of strong & weak interactions which can be calculated with (due to asymptotic freedom of QCD) — at any rate at short distances.

5.) Moral for HSRP is never to abandon a programme too hastily in the ~~lift~~ light of
a) inconsistency (the infinities)
b) lack of novel prediction due to the Computation gap.

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Concluding remarks

In a sense nothing has happened in QFT since around 1930. In Kuhnian terms there has been no revolution, no paradigm switch (But cf the rival S-Matrix programme in the 1960s)[*] — just a gradual working out of the immense and really unsuspected technical richness of the theory.
But in respect of the interpretation of QFT what the formalism really commits us to, ontologically speaking, there has been too much rhetoric and too little informed and critical discussion. The object of this paper has been to entice (and assist) philosophers who would like to join in the arguments.